Eulerian/Lagrangian Sharp Interface Schemes for Multimaterials

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Physical and engineering problems that involve several materials are ubiquitous in nature and in applications. The main contributions in the direction of simulating these phenomena go back to [1] for the model and [2] for numerical simulations. However, the numerical scheme presented in that paper is relatively complicated and has the disadvantage that the interface is diffused over a certain number of grid points. We propose a simple second-order accurate method to recover a sharp interface description keeping the solution stable and non-oscillating. This scheme can be adapted to both eulerian and lagrangian frameworks.

The conservative form of elastic media equations in the eulerian framework are

$$\begin{cases} \rho_t + \mathrm{div}_x(\rho u) = 0 \\ (\rho u)_t + \mathrm{div}_x(\rho u \otimes u - \sigma) = 0 \\ (\rho e)_t + \mathrm{div}_x(\rho e u - \sigma^T u) = 0 \\ (\nabla_x Y)_t + \nabla_x(u \cdot \nabla_x Y) = 0 \end{cases} \tag{1}$$

The unknowns are the density $\rho(x,t)$, the velocity $u(x,t)$, the total energy per unit mass $e(x,t)$ and the backward characteristics of the problem $Y(x,t)$. Here $\sigma(x,t)$ is the Cauchy stress tensor in the physical domain. The counterpart of these equations in the lagrangian framework are

$$\begin{cases} (\rho_0 X_t)_t - \mathrm{div}_\xi(\mathcal{T}) = 0 \\ (\rho_0 e)_t - \mathrm{div}_\xi(\mathcal{T}^T X_t) = 0 \end{cases} \tag{2}$$

The unknowns are the velocity $X_t(\xi,t)$ (the direct characteristics of the problem are $X(\xi,t)$) and the total energy per unit mass $e(\xi,t)$. Here $\mathcal{T}(\xi,t)$ is the first Piola-Kirchoff stress tensor in the reference domain and ρ_0 is the initial density.

To close the system, a constitutive law is chosen:

$$\varepsilon = e - \frac{1}{2}|u|^2 = \frac{\exp\left(\frac{s}{c_v}\right)\ \rho^{\gamma-1}}{\gamma-1} + \frac{p_\infty}{\rho} + \frac{\chi}{\rho_0}(\mathrm{Tr}(\overline{B}) - 2) \tag{3}$$

where $s(x,t)$ is the entropy and $\overline{B}$ is the modified left Cauchy-Green tensor which depends on $\nabla_x Y$. The constants $c_v, \gamma, p_\infty, \chi$ characterize a given material. The two first terms of (3) represent a stiffened gas and the third one represent a Neo-Hookean elastic solid. The stress tensors σ and $\mathcal{T}$ are then derived from this constitutive law as a function of the problem unknowns.

We have already developped a numerical scheme to solve the eulerian equations of conservation (1). This scheme is based on a directional splitting on a fixed cartesian mesh where the fluxes are computed by an HLLC approximate Riemann solver. The interface is kept sharp by using a non-conservative numerical flux for the computational cells that are crossed by the contact discontinuity. We refer for the 1D numerical results to [3] for the first order scheme and [4] for second order scheme.

First of all, in the proposed presentation we will provide new numerical results of fluid-structure interaction involving 2D impacts. The initial configuration is a "T" shape (filled with copper or water) and surrounded by air. We impose an initial horizontal velocity on the left part of the "T" shape. In Fig.1 we present the numerical results obtained with our multimaterial scheme of air-copper (left) and air-water (right) media after the impact.

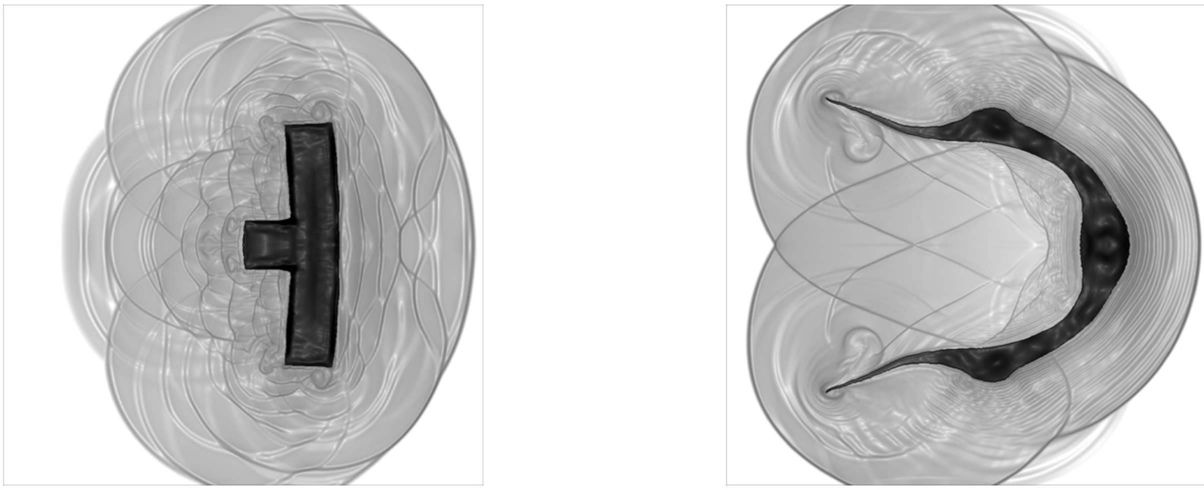

Figure 1: Schlieren image of the density for the air-coppper (left) and air-water (right) after the impact.

In addition, an ongoing work is to adapt our HLLC scheme to the lagrangian equations of conservation (2). We plan to compare the results with those of Fig.1 and develop additional fluid-structure test cases with free surfaces (explosions and detonations applications) which can not be considered in the eulerian frame.

References

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